Nonlinear Ovalization Analysis of Thin-walled Circular Pipes in Generalized Beam Theory

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ABSTRACT

Development of nonlinear Generalized Beam Theory (GBT) analysis was first attempted by Schardt [1, 2] and his co-workers [3] to solve coupled stability problems. Here, the linear uncoupled combination of deformation modes in linear GBT analysis [4] was extended to a nonlinear analysis by introducing the concept of third order deformation modes coupling tensor ${}^{ijk}\mathbb{X}$ which indicates if there is a coupling among the internal forces generated in mode **j** due to a virtual displacement in mode **k** and an initial stress in mode **i**.

In this study the formulation of fully geometrically nonlinear GBT analysis is developed for thin-walled circular pipes. This formulation uses the Sanders [5] definition of kinematic relations for circular cylindrical shells which satisfies the Love-Kirchhoff assumption. Here, the nonlinearity is only considered for the membrane strains since for thin-walled sections plate contributions are insignificant. Furthermore displacements are generally kept small since the formulation is based on a Total Lagrangian description. The initial second-order and third-order stress and displacement tangent stiffness matrices were developed using third-order (${}^{ijk}\mathbb{X}$) and fourth-order (${}^{ijkl}\mathbb{X}$) GBT mode coupling tensors.

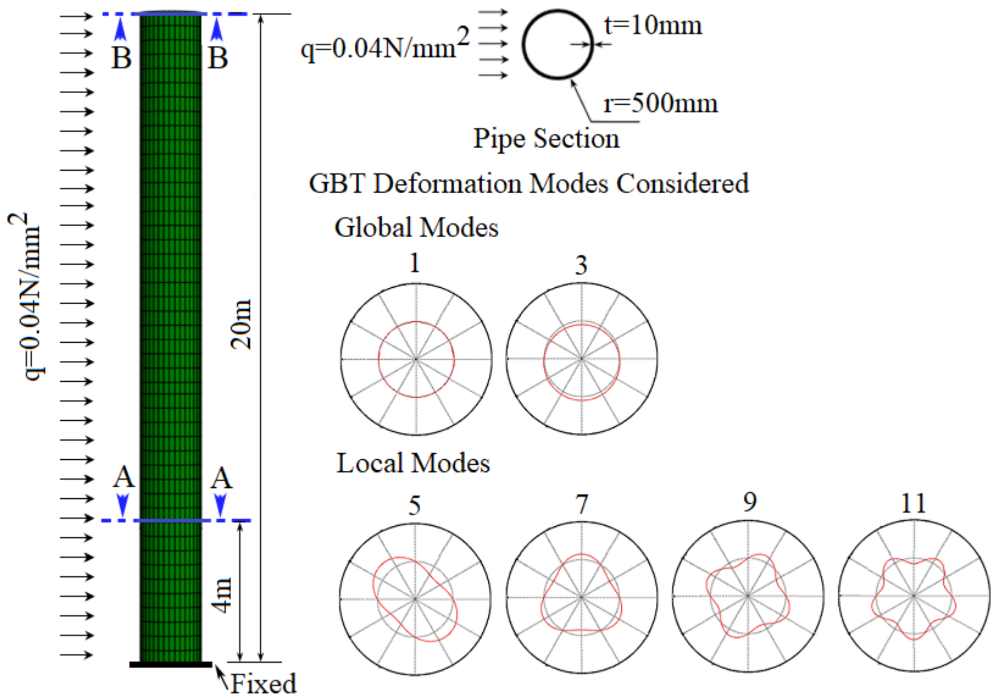
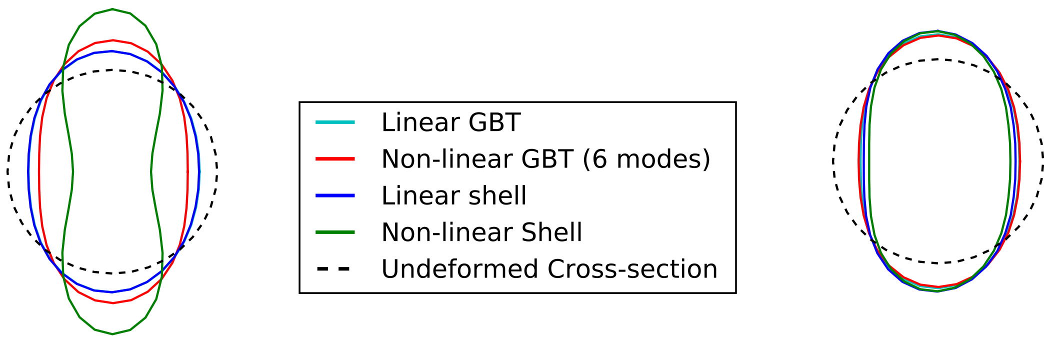


Figure 1: A cantilever cylindrical pipe example.

In the example presented (Figure 1), only the nonlinearity in longitudinal strain is considered due to the null transverse extension and membrane shear strain assumptions of GBT. The purpose of this example is to demonstrate the formation of large ovalization due to the increase in bending moment at the lower section of the pipe which leads to kink formation and eventually failure. Here, the global tip displacement for the given loading is $1m$. At this displacement the linear and nonlinear analysis of GBT and shell finite element model are compared in figure 2 at two sections. The local deformation at section **B** of the pipe doesn't show any significant difference for linear and nonlinear analysis of the GBT and finite shell element models in general. However, at section **A** there is a significant difference between nonlinear results of the GBT and finite shell element models, which is caused by the missing nonlinearity due to the null transverse and shear strain assumptions of GBT. Currently studies are underway to improve these results by considering additional shear deformable GBT modes. Note that, in figure 2 the linear analysis results of the GBT and finite shell element models are identical throughout the cross-section and length of the pipe.



(a) Section **A** (b) Section **B**

Figure 2: Comparison of local deformations between GBT and Shell results (scaled by 20)

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REFERENCES

[1] Richardt Schardt. Generalized beam theory - an adequate method of coupled stability problems. *Thin-Walled Structures*, 19:161–180, 1994.

[2] Richardt Schardt. Lateral torsional and distortional buckling of channel-and hat-sections. *Journal of Constructional Steel Research*, 31:243–265, 1994.

[3] Leach.P Heiny D. Davies, J.M. Second-order generalised beam theory. *Journal of Constructional Steel Research*, 31:221–241, 1994.

[4] Richard Schardt. *Verallgemeinerte technische Biegetheorie: Lineare Probleme*. Springer-Verlag, 2013.

[5] J Lyell Sanders Jr. Nonlinear theories for thin shells. *Quarterly of Applied Mathematics*, 21(1):21–36, 1963.